

TRADING FREQUENCY AND IMPLIED TRANSACTION COSTS OF FOREIGN EXCHANGE OPTIONS

Shmuel Hauser,^{*} Azriel Levy,[†] and Uzi Yaari[‡]

I. INTRODUCTION

The ability to create a perfect hedge, the basis for risk-neutral options valuation models, hinges on the absence of transaction costs. This is because a perfect hedge requires a continuous adjustment of the portfolio replicating the option, pushing transaction costs to infinity. Several authors, including Hodges and Neuberger (1990), Gilster and Lee (1984), Leland (1985), Hauser and Levy (1991), and Rictkhen and Kuo (1988), show that less frequent portfolio adjustments with finite transaction costs allow only an imperfect hedge, leading to an ambiguous option

^{*}Israel Securities Authority, Jerusalem, and Bar-Ilan University, Ramat Gan.

[†]Bank of Israel, and Hebrew University, Jerusalem.

[‡]Rutgers University, Camden.

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price within upper and lower bounds. But Leland (1985) and Figlewski (1989) argue that transaction costs and other imperfections in the real markets may be so great that the standard arbitrage model can only establish impractically wide price bounds.

As shown by Hodges and Neuberger (1990), one way of dealing with the hedging errors caused by discrete trading is to employ a utility-dependent hedging strategy. In an alternative method, Leland (1985) modifies the Black–Scholes strategy to include transaction costs and a trading time interval. This method relies on the assumption that hedging errors are uncorrelated with the market for the underlying asset, implicitly relying on additional assumptions regarding the pricing of that asset. Hodges and Neuberger (1990) argue that since exact replication of the option is impossible at finite costs, Leland's method is not optimal. In response, they develop a replication strategy based on a loss function and restricted by investor risk preferences.

This paper develops a model for pricing options on foreign currencies, a binomial model that includes transaction costs and treats the trading interval as endogenous, but is not dependent on investor risk preference. Our inclusion of transaction costs in a discrete arbitrage model resembles a recent paper by Boyle and Vorst (1992). However, those authors do not address the issue raised by Leland (1985) and Figlewski (1989) of whether the strategy of portfolio adjustment is consistent with real-life transaction costs. We address this issue by demonstrating that the upper and lower bounds of currency option prices are fairly narrow under feasible unit transaction costs. This result is based on a scenario where the investor is faced at each point in time with a trade-off between bearing the cost of hedging errors or increasing transaction costs. Given the cost per transaction, more frequent trading would lead to a smaller hedging error (at the limit, a single price) but larger total transaction costs. These factors would have conflicting effects on the spread between the upper and lower price bounds. The low transaction costs implied by our estimation suggest that the market for currency options is dominated by institutional investors.

II. THE MODEL

For an institutional investor operating in the interbank market for foreign currency options, the main source of transaction costs is the bid–ask foreign exchange spread.

- Let S_{tB} , S_{tA} , S_t : respectively bid, ask, and middle exchange rates in period t ($t = 0, 1, \dots$), measured in dollars per unit of foreign currency;
 r , r^* : 1 plus domestic and foreign interest rates, respectively;
 X : strike price of the call option.

Denoting $\alpha^2 = S_{iA}/S_{iB}$, where $(\alpha^2 - 1)$ is the percentage difference between the bid and ask prices of the underlying currency, we define the *middle* price as the geometric average of the bid and ask prices

$$S_i = (S_{iB} \cdot S_{iA})^{1/2}, \quad (1)$$

where $S_{iB} = S_i/\alpha$ and $S_{iA} = S_i\alpha$.

We further assume that the rate of return of the underlying asset's middle exchange rate is distributed binomially with n trading periods to maturity, where the two states are denoted by u and d . We also assume that the proportional unit transaction costs α are constant.

A. Upper Price Bound

To find the call option's upper price bound C at time 0, we construct the following arbitrage transaction. The investor purchases an amount δ of the foreign currency at the ask price and sells short one unit of the option at the bid price, financing the transaction by borrowing dollars. In the transaction described in Table 1, C_u and C_d denote the values of the call option in period 1 in states u and d , respectively.

To find the hedge ratio δ , we equate the return in the two states of nature

$$\delta S_{ur^*}/\alpha - C_u = \delta S_{dr^*}/\alpha - C_d$$

and solve for δ :

$$\delta = [\alpha(C_u - C_d)]/[Sr^*(u - d)]. \quad (2)$$

To prevent arbitrage, the return in states u and d must be nonpositive, indicating

$$\delta S_{ur^*}/\alpha - C_u - (\delta S\alpha - C)r \leq 0. \quad (3)$$

Table 1. Arbitrage Transaction for Upper Bound

Transaction	Time 0	Time 1	
		State u	State d
1. Buy δ units of currency, sell option	$-\delta S\alpha + C$	$\delta S_{ur^*}/\alpha - C_u$	$\delta S_{dr^*}/\alpha - C_d$
2. Borrow at \$ borrowing rate	$\delta S\alpha - C$	$-(\delta S\alpha - C)r$	$-(\delta S\alpha - C)r$

Substitution of (2) in (3) yields the following solution for the upper price bound of a call option at time 0:

$$C \leq [PC_u + (1 - P)C_d]/r, \quad (4)$$

where $P = [\alpha^2 r - r^* d]/[r^*(u - d)]$ and $0 \leq P \leq 1$.

We now turn to the derivation of the n -period upper bound. In period 1 there are two possible states of nature: $S_1 = Su$ or $S_1 = Sd$, where S is the initial price of the underlying currency. For $S_1 = Su$ and $C_1 = C_u$, we can repeat the arbitrage described in Table 1 to find the upper price bounds

$$C_u \leq [PC_{uu} + (1 - P)C_{ud}]/r, \quad (5)$$

$$C_d \leq [PC_{du} + (1 - P)C_{dd}]/r, \quad (6)$$

where C_{uu} , C_{ud} , C_{du} , and C_{dd} are the option prices in period 2 in each of the four possible states of nature.

To find the two-period upper bound, we substitute (5) and (6) into (4):

$$C \leq [P^2 C_{uu} + 2P(1 - P)C_{ud} + (1 - P)^2 C_{dd}]/r. \quad (7)$$

To obtain the n -period upper bound, we repeat the same procedure in an iterative manner, obtaining

$$C \leq \frac{1}{r^n} \sum_{j=0}^n \binom{n}{j} P^j (1 - P)^{n-j} C(u, j)/\alpha, \quad (8)$$

where state u occurs j times, state d occurs $n - j$ times, and the option value at expiration is

$$C(u, j)/\alpha = \text{Max}[0, Su^j d^{n-j}].$$

Note that the option will be exercised in period n if

$$Su^j d^{n-j}/\alpha - X \geq 0. \quad (9)$$

The first term on the left-hand side of (9) is the bid currency price. The arbitrage process that leads to the upper price bound requires purchasing the underlying asset and selling the option. If the option is exercised, the writer performing the arbitrage would deliver the underlying asset, which could otherwise be sold for the bid exchange rate, S_n/α . Thus, (9) is the appropriate cash flow for the boundary condition in case the option is exercised.

B. Lower Price Bound

To find the call option's lower price bound, we describe in Table 2 an arbitrage transaction where the investor sells δ' of the foreign currency at the bid rate, buys one unit of the option at its ask price, and lends dollars at the dollar lending rate.

To find the hedge ratio δ' , we equate the return in the two states of nature

$$-\delta' S u^* \alpha + C_u = -\delta' S d r^* \alpha - C_d \quad (10)$$

and solve for δ' :

$$\delta' = [(C_u - C_d)] / [S r^* (u - d) \alpha]. \quad (11)$$

Table 2. Arbitrage Transaction for Lower Bound

Transaction	Time 0	Time 1	
		State <i>u</i>	State <i>d</i>
1. Sell δ' units of currency, buy option	$\delta' S / \alpha - C$	$-\delta' S u^* \alpha + C_u$	$-\delta' S d r^* \alpha + C_d$
2. Lend at r lending rate	$-\delta' S / \alpha + C$	$(\delta' S / \alpha - C) r$	$(\delta' S / \alpha - C) r$

As in the case of the upper bound, the return in both states must be nonpositive. Solving the inequality, we derive the following lower bound:

$$C \geq [P' C_u + (1 - P') C_d] / r, \quad (12)$$

where $P' = [r - r^* d \alpha^2] / [r^* (u - d) \alpha^2]$ and $0 \leq P' \leq 1$. We continue the iterative process to derive the n -period lower bound

$$C \geq \frac{1}{r^n} \sum_{j=0}^n \binom{n}{j} P'^j (1 - P')^{n-j} C(u, j) \alpha, \quad (13)$$

where state u occurs j times, state d occurs $n - j$ times, and the value of the option in period n is $C(u, j) = \text{Max}[0, S u^j d^{n-j}]$. In creating the arbitrage process that leads to the lower bound, the investor shorts (sells) the underlying asset and buys the call option. In period n , he or she must buy the underlying currency and pay the ask price $S_n \alpha$. Therefore, the arbitrageur will exercise the call option if

$$S u^j d^{n-j} \alpha - X \geq 0. \quad (14)$$

III. SIMULATION

This section is divided into two parts. First, we illustrate the binomial valuation model allowing for the presence of transaction costs. We then combine our arbitrage-based theoretical model with empirical data on foreign currency option prices to derive implied transaction costs. Our estimates allay the fear expressed by Leland (1985) and Figlewski (1989) that transactions costs may be too high to accommodate an arbitrage approach to option pricing by leading to very wide bounds of option prices. Those estimates can also serve as a basis for deciding the

question raised by Galai (1990): Who is the dominant or marginal trader determining market prices?

A. Illustration of the Model

Our illustration of the transaction-based binomial model uses the following parameters: $S = 100$, $X = 100$, $T = 0.25$, $\sigma = 0.15$, $r = r^* = 0.1$, $u = \exp(\sigma\sqrt{dt})$, and $d = 1/u$. Note that parameters u and d are defined in a manner ensuring that our model converges on the Black-Scholes option valuation model in the absence of transaction costs (see Cox and Rubinstein, 1985). Using these parameters, we calculate the upper and lower price bounds of the currency call option for different transaction costs under various trading intervals.

The following results can be garnered from Figure 1. First, as expected, the higher the transaction costs, the greater is the spread between the upper and lower price bounds, namely, the greater is the range over which arbitrage profits are impossible. Second, given the cost per transaction, the greater the number of trading intervals within a given time to maturity, the wider is the range between the upper and lower price bounds. The common assumption that the binomial process is a good proxy for the Wiener continuous stochastic process is no longer valid when transaction costs are present. Third, the observed effect of the trading frequency on price bounds indicates the need to choose a frequency that would consider two types of transactions costs. Any attempt to save on such costs by lowering the frequency of trading would result in higher costs due to a less perfect hedge and a wider spread of price bounds (see Hauser and Levy, 1991).

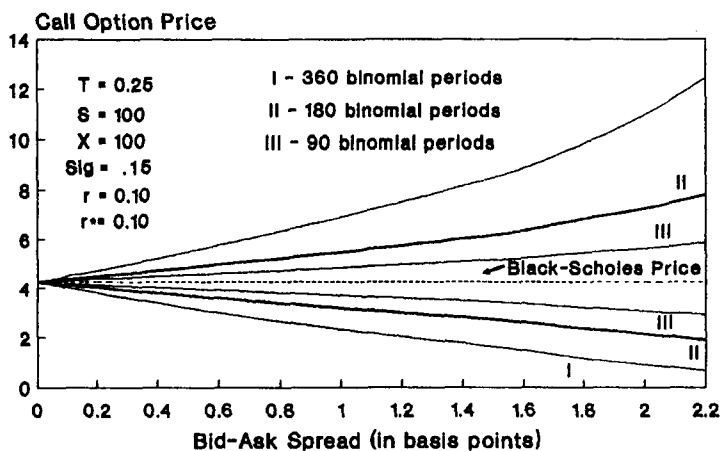


Figure 1. Upper and lower bounds of foreign currency options with transaction costs

B. Results

The data used represent a sample of transactions of currency call options on the Philadelphia Stock Exchange from March 1983 to December 1985. Compiled by the exchange, the data consist of synchronous option prices and spot exchange rates. There are 5165 observations covering five currencies: West German mark (GM), Japanese yen (JY), Swiss franc (SF), and British pound (BP). The risk-free interest rates of these countries are approximated by the 90-day Eurocurrency deposit rates. Note that, over the sample period, interest rates in Germany, Japan, and Switzerland were considerably lower than those in the United States, allowing the use of European-call option valuation models to price American call options. This is because in cases where foreign interest rates are lower than the domestic ones, the probability of early exercise is practically zero, and pricing errors are insignificant (see Shastri and Tandon, 1986).

The implied unit transaction costs (hereafter ITC) measured by α_t are estimated by α_t in the equation

$$CMKT_t = CMOD_t(\alpha_t, S_t, X, T, u, d, r_t, r_t^*),$$

where $CMKT_t$ is the observed call value and $CMOD_t$ the calculated call value based on a set of observed parameters and an initial guess of α_t . Following Cox and Rubinstein (1985) and others, we assume $u = \exp(\sigma\sqrt{dt})$ and $d = 1/u$, where σ is the standard deviation estimated by the historical annualized logarithmic returns. Note that under this assumption and in the absence of transaction costs, the n -period binomial model converges on the Black-Scholes model, implying that the Black-Scholes price lies between the upper and lower price bounds of the model. The ITC is calculated by equating $CMKT_t$ with the call upper price bound when $CMKT_t$ is higher than the Black-Scholes price. Equality with the lower price bound is assumed when $CMKT_t$ is lower than the Black-Scholes reference price. In each case, the ITC is estimated under two alternative assumptions concerning the trading frequency: once a day and four times a day. Thus, the trading frequency over the life of the option would be n or $4n$, respectively, where n is the number of days to maturity.

The results displayed in Table 3 reveal unit transaction costs of between three and five basis points under daily trading, and a quarter of these rates under four-times-a-day trading. The interpretation of these results should be approached with caution. Although the bid-ask spread in the theoretical model is the only cost of transaction, our estimation of the implied spread is likely to be affected by other transaction costs, such as the bid-ask spreads of foreign and domestic interest rates. The emerging differences in the relative bid-ask spread among the four currencies are likely to be inflated due to this factor.

Based on a procedure setting the market price equal to the upper or lower price bound, our results have important implications. First, our procedure of estimating

Table 3. Implied Transaction Costs in Foreign Currency Options with Different Trading Intervals (March 1983–September 1985)^a

<i>Trading interval</i>	<i>Currency</i>			
	<i>DM</i>	<i>JY</i>	<i>SF</i>	<i>BP</i>
<i>n</i>	1294	1283	1279	1319
<i>Once a day</i>				
Average	4.547	2.901	4.148	3.686
S.E. ^b	0.108	0.077	0.104	0.087
Median	3.406	2.019	2.868	2.889
<i>Four times a day</i>				
Average	1.164	0.718	1.049	0.925
S.E. ^b	0.030	0.019	0.028	0.022
Median	0.843	0.503	0.717	0.718

Notes: ^aImplied transaction costs are measured as 100 times the bid-ask spread (2α) as a proportion of the exchange rate (dollars per foreign currency units).

^bS.E. is the standard error of the implied transaction costs.

transaction costs circumvents the question of frequency of trading since the cost per day is independent of that frequency. This allows us to estimate the ITC without information about the number of transactions per day. Second, our estimated low transaction costs may be interpreted as evidence that currency option pricing is dominated by large institutional investors. Third, contrary to the prevailing view, our risk-neutral model with discrete portfolio adjustments does not push transaction costs to a prohibitive level. For example, a cost of 0.04% per day on a 90-day option is translated into a total cost of 3.7% over the life of the option. These finite “small” costs are consistent with an investor’s choice of trading frequency that leads to equality between the marginal cost of hedging errors and the marginal gain from reducing transaction costs.

IV. SUMMARY

This study explores the viability of arbitrage in determining the price of foreign exchange options when adjustments of the replicating asset portfolio are subject to transaction costs. To this end, we first develop a model for valuing those options and then use empirical data and simulation to calculate the bid-ask spread consistent with various levels of transaction costs and trading frequencies. Our findings show that the price spread implied by feasible cost and trading interval parameters is fairly narrow, indicating a role for arbitrage in that market. Our estimated low transaction costs suggest that the market for currency options is dominated by institutional investors.

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